Lesson 7.3--Sum and Difference Identities

4 new identities:

$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y)$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

These are used to find the exact value of expressions such as sin15°, cos75°, tan165°, etc.

The key is that 15° = 45° - 30°,
75° = 45° + 30° and
165° = 120°+45°
165° = 210° - 45°

The proof of these identities is on page 437...IF you're interested!

Can you find sin165° using our new identity?!

examples-->>

Can you find sin165° using our new identity?!

$$\sin 165^\circ = \sin(135^\circ + 30^\circ)$$

$$= \sin 135 \cdot \cos 30^\circ + \cos 135^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

Find the exact value of sin 435°

$\sin 435° = \sin(435°-360°) = \sin 75°$ (coterminal anlges)

$\sin 75° = \sin(30°+45°)$

$= \sin 30°\cos 45° + \cos 30°\sin 45°$ (sum identity for sine)

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ (chapter 5 stuff!)

$= \frac{\sqrt{2} + \sqrt{6}}{4}$ (simplify)

If this had been cos435°, you'd do the same thing....just use the cosine identity: $\cos 75° = \cos(30°+45°)$

$= \cos 30°\cos 45° - \sin 30°\sin 45°$..... (notice the sign!)

EX: Find the exact value of tan 255°

$$\tan 255° = \tan(300° - 45°)$$

$$= \frac{\tan 300° - \tan 45°}{1 + \tan 300° \tan 45°}$$

see trig id for tan(x - y)

$$= \frac{-\sqrt{3} - 1}{1 + (-\sqrt{3})(1)} = -\frac{\sqrt{3} - 1}{1 - \sqrt{3}}$$

multiply by $\frac{1 + \sqrt{3}}{1 + \sqrt{3}}$ to simplify

$$= \sqrt{3} + 2$$

example: Find the exact value of $\sin(x-y)$ if $0<x<\pi/2$ and $0<y<\pi/2$ AND $\sin x = 9/41$ and $\sin y = 7/25$

This is really easier than it seems!

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$= (9/41)(?) - (?)7/25$

(We just need to find cosy and cosx)

But HOW?!
(use other trig ids!)

example: Find the exact value of $\sin(x-y)$ if $0<x<\pi/2$ and $0<y<\pi/2$ AND
$\sin x = 9/41$ and $\sin y = 7/25$

This is really easier than it seems!

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$= (9/41)(?) - (?)7/25$

(We just need to find cosy and cosx)

$\sin^2 y + \cos^2 y = 1$
$(7/25)^2 + \cos^2 y = 1$
...
$\cos y = 24/25$

$\sin^2 x + \cos^2 x = 1$
$(9/41)^2 + \cos^2 x = 1$
...
$\cos x = 40/41$

now just substitute and do the math!

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$= (9/41)(24/25) - (40/41)7/25$

$= -64/1025$

Verify: $\csc(3\pi/2 + \theta) = -\sec\theta$

$$\frac{1}{\sin(3\pi/2+\theta)} = -\sec\theta$$

$$\frac{1}{\sin 3\pi/2 * \cos\theta + \cos 3\pi/2 * \sin\theta} = -\sec\theta$$

$$\frac{1}{\underbrace{\sin 3\pi/2}_{-1}\cdot\cos\theta + \underbrace{\cos 3\pi/2}_{0}\cdot\sin\theta} = -\sec\theta$$

$$\frac{1}{-\cos\theta} = -\sec\theta$$

$$-\sec\theta = -\sec\theta$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

STRATEGY: List special angles in radian measure to determine sum/difference

$$\frac{\pi}{2} = \frac{6\pi}{12}$$

$$\frac{\pi}{6} = \frac{2\pi}{12}$$

$$\frac{\pi}{3} = \frac{4\pi}{12}$$

$$\frac{\pi}{4} = \frac{3\pi}{12}$$

$$\pi = \frac{12\pi}{12}$$

$$\frac{3\pi}{2} = \frac{18\pi}{12}$$