§4.2 - Linear & Angular Velocity

see p. 352

angular displacement
angular velocity
linear velocity

angular displacement → change in placement of angle

Ex. Determine the angular displacement in radians of 4.5 revolutions. Round to the nearest tenth.

1 REV. = 2π rads.

so 4.5 REV. = $4.5 * 2\pi$ or 28.3 rads

Angular velocity is the ratio of the angular displacement, θ, to the time, t, required by that displacement. (see p. 352)

$$\omega = \frac{\theta}{t}$$ (θ in radians)

angular velocity "omega"

(θ = # revs. * 2π)

compare to

$d = r \cdot t$

$\frac{d}{t} = r$

change in position

$\frac{\theta}{t} = \omega$

angular velocity is the rate at which the angle is changing... always in radians per time.

NOTE: angular velocity is NOT dependent on the distance (radius) from the center of the rotating object.

$\boxed{\text{EX2}}$ Determine the angular velocity if 7.3 revolutions are completed in 5 seconds.

$$\omega = \frac{\theta}{t} \longleftarrow \theta = 7.3 * 2\pi \text{ (ang. disp.)}$$

$$\omega = \frac{7.3 * 2\pi}{5}$$

$$\omega \approx 9.2 \text{ rad/sec}$$

DIMENSIONAL ANALYSIS

Treat units of measure as factors when multiplying (they cancel out)

LOOK @ PRBLM ON TOP OF P. 352

ang. vel. = $\omega = \frac{\theta}{t}$

$\boxed{\text{Find } \omega \text{ in rads/sec}}$

$$\omega = \left(2.625\,\frac{\text{rev}}{\text{min}} \cdot 2\pi\,\frac{\text{rads}}{\text{rev}}\right) = 16.49\,\frac{\text{rad}}{\text{min}} \cdot \frac{1\text{min}}{60\text{sec}}$$

$$= 0.275\,\frac{\text{rad}}{\text{sec}}$$

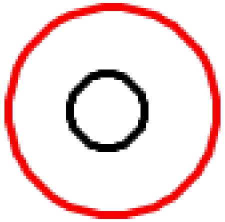
(see ex. 3)

UDO:

A circular serving table in a buffet has a radius of 3 feet. It makes 2.5 revolutions per minute. Determine the angular velocity in radians per second of a bowl of peaches sitting on the serving table.

$$w = \frac{\theta}{t}$$

$$w = \frac{2.5\text{ rev}}{1\text{ min}} \cdot \frac{2\pi\text{ rads}}{1\text{ rev}} = \frac{15.707\text{ rad}}{1\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} = .262\text{ rads/sec}$$



Linear Velocity:

Who travels farther on a carousel? The person on the inner or outer circle? How much?

$d = r \cdot t$ (distance)

$s = v \cdot t$ (arc distance)

$\frac{s}{t} = v$ (linear velocity)

$\frac{r\theta}{t} = v$ ($s = r\theta$ ... §6.1)

$v = \frac{r\theta}{t}$ (LINEAR VELOCITY)

recall:

θ = ang. disp

$\theta = (\#\text{revs})(2\pi)$

or $v = r\omega$ ($\omega = \frac{\theta}{t}$ ... ang. vel.)

(NOTE: Linear velocity is always distance per time)

EX. 5

Determine the linear velocity of each rider on the carousel...

$v = (\text{radius})(\text{ang. vel.})$ — $r \cdot w$

$V = (11)(0.275)$ — $w = \frac{(2\frac{5}{8})(2\pi)}{60}$

$V = 3.025$ FT/sec

$v = r \cdot w$

$v = 20(0.275)$

$v = 5.5$ FT/sec.

Summary:

angular displacement = (# rev)(2π)

angular velocity (ω) = $\frac{\theta}{t}$

linear velocity: $v = r \cdot \frac{\theta}{t}$

r = radius ; $\frac{\theta}{t}$ = ang. velocity

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angular displacement = change in $\angle$

ang. velocity = rate @ which $\angle$ changes

linear velocity = rate of change of a "dot" on $\odot$

EX.6. The tires on a race car have a diameter of 30 inches. If the tires are turning at a rate of 2000 revolutions per minute, determine the race car's speed in miles per hour.

$v = r \cdot \omega$

$\omega = \frac{\theta}{t}$

$\theta = \angle$ disp.

$= (2000)(2\pi)$

$= 4000\pi$

$r = \frac{1}{2}(30) = 15$ in.

$v = (15\text{ in.})\left(\frac{4000\pi}{1\text{ min}}\right) = 188495.6$ in/min

want mph

$188495.6 \frac{\text{in}}{\text{min}} \cdot \frac{60\text{ min}}{1\text{ hr.}} \cdot \frac{1\text{ FT}}{12\text{ in.}} \cdot \frac{1\text{ mile}}{5280\text{ FT}} = 178.5$ mph

Work each problem, rounding to the tenth.

1. Determine the angular displacement in radians of 471 revloutions.

2. Determine the angular displacement in radians of 9.3 revolutions

3. Determine the angular velocity if 450 revolutions are completed in 12 minutes.

4. Determine the angular velocity if 7.6 revolutions are completed in 11 seconds.

5. Determine the linear velocity of a point rotating at an angular velocity of 75 radians per second at a distance of 40 inches from the center of the rotating object.

1. 2959.4 rads; 2. 58.4 rads; 3. 235.6 rads/min;
4. 4.3 rads/sec; 5. 3000 in/sec

At Jubilee Gardens in London, England, the British Airways London Eye Ferris wheel opened in January, 2000. It was built for the new millennium. The diameter of the wheel is 443 feet, its highest elevation above the ground is 446 feet 7 inches, and from the top, yuou can see for 30 miles. Suppose at a particular time the operator controls the speed so that the wheel revolves once every 7.5 minutes.

a. Determine the angular velocity for a rider in radians per second

b. Determine the linear velocity for a rider.

a. The Ferris wheel makes 1/7.5 revs per min.

$$\frac{1 \text{ rev}}{7.5 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{2\pi \text{ rads}}{1 \text{ rev}} = 0.014 \text{ rads/sec}$$

Each rider has an angular velocity of about 0.014 rads/sec

b. $v = r\omega$

$v = 221.5(0.014)$

$v = 3.1$ ft/sec

The linear velocity of a rider is about 3.1 ft/sec