PC Lesson 3.1--EXPONENTIAL FUNCTIONS AND THEIR GRAPHS

PAST: Linear, quadratic, absolute value, radical, polynomial, rational (algebraic)
PRESENT: exponential and logarithmic (transcendental)
FUTURE: trigonometric (transcendental)

All are "special" functions that we study b/c they model many real life situations

$f(x) = a^x$, where $a > 0$, $a \neq 1$ and x is any real number

Domain:
Range:
Increasing:
Decreasing:
Intercepts:
Asymptote(s):

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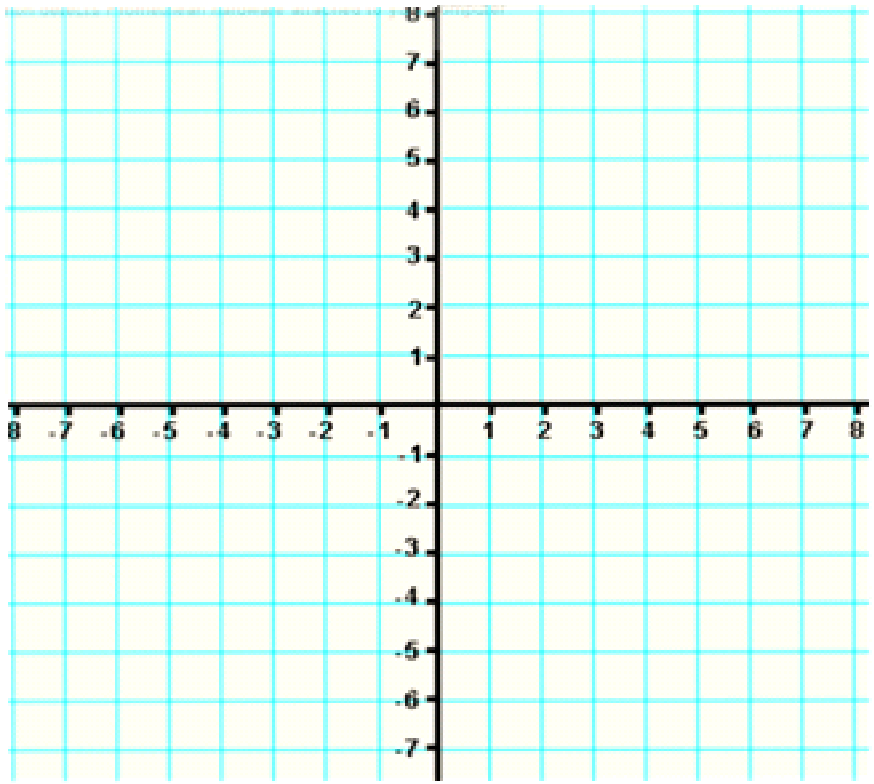


$g(x) = f(x) \pm c$

$h(x) = f(x \pm c)$

$k(x) = f(-x)$

$p(x) = -f(x)$

What about $y = \frac{1}{2}^{x}$? ummmmm?! What would this function look like?!

Could you possibly rewrite it in a different form?!

I believe you could (or at least I could!) WHY?

SO, if the "a" value is between 0 and 1 ($0<a<1$) then the graph will essentially be reflected over the y-axis! Isn't that exciting?! Doesn't it make you want to sing and shout?! Well, maybe not ➢➢➢➢➢➢➢

So $f(x) = 2^X$ then $g(x) = f(-x)$ is the same as $g(x) = 2^{-X}$ is the same as $g(x) = ½^X$

Which of the following functions below are equivalent? (sample hw question)

(1) $g(x) = 5^{x-2}$ $f(x) = 5^X - 25$ $h(x) = 1/25(5^X)$ (Careful, this requires finagling!)

(2) $g(x) = \frac{1}{9}(3^{-X})$ $f(x) = (1/3)^{x-2}$ $h(x) = 9(3^{-X})$

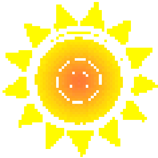
The Natural base *e*

Often real life applications involve the number *e*. This is an irrational number (like π) and is known as the natural base.

It is often used when dealing with *continuous* growth or decay...like compound interest, population growth or *natural* decay/growth in *nature* (NOTE: mathematics *really* happens....you can't deny it!) (see ex. 8 - 11 for real life apps.)

"From where does it come?" you might ask. It can be approximated by

$$(1 + 1/x)^x \text{ as } x \rightarrow \infty$$



Let's just see how this works....

In your GUT type in $Y1 = (1 + 1/x)^{x}$ and

set your table (tblset) to start at a really big number....pick 10000.
set Δtbl to 1000.

Now make your table increase to very large numbers (use arrow key)

What is the y-value approaching? This is the approximation for the number *e*!

(e is approximately 2.72)

NOW, you treat this base like any other number....it's just a number (but a very special number)

$f(x) = e^x$ is just another exponential function (with the base being approximately 2.72)

How would it compare to the graph of $y = 2^x$ or $y = 3^x$?

What would $g(x) = f(-x)$ look like? What about $h(x) = -f(x)$? And let's not forget $k(x) = f(x) - 3$?

Find this number on your calculator(s). You'll probably have two....a plain *e* and then an e^x

You will most likely use e^x more b/c you will usually raise e to a power. (see ex. 6 on pg 221)

Practice: Evaluate: (a) e^{-2} (b) e^{-1} (c) e^{2} (d) e^{3} (e) e^{1}

You should be able to do this with both calculators!

Two important formulas that are used to determine compound interest:

$A = P(1 + r/n)^{nt}$ used when compounding interest n times per year

$A = Pe^{r\,t}$ used when compounding interest *continuously*

"r" is interest rate

"t" is the time (in years)

"P" is the amount of principal invested

"A" is the balance in the account after t years

"n" it the number of times the interest is compounded yearly

NOTE: The derivation of this formula is on page 222...I find it bores most students so I don't discuss it. (Your welcome!)

Example:

A sum of $9000 is invested at an annual interest rate of 8.5% for 30 years.

Find the balance if the interest is compounded annually

Find the balance if the interest is compounded monthly

Find the balance if the interest is compounded continuously

Population Growth Example

The approximate number of fruit flies in an experimental population after t hours is

$$Q(t) = 20e^{0.03t}, \ t \geq 0$$

(a) Find the initial number of fruit flies in the population

(b) How large is the population of fruit flies after 52 hours

(c) Use your GUT to determine when the number of fruit flies will be 160