

Lesson 5.4---Exponential Functions Differentiation & Integration

NOTE: The first 2 pages of this lesson is precal...you're on you own with that!

$$\frac{d}{dx}(e^x) = e^x$$

NOTE: Proof of this is on page 343

$$\frac{d}{dx}(e^u) = e^u \cdot u'$$

$$\frac{d}{dx}(e^{3x^2}) = e^{3x^2} \cdot 6x = 6xe^{3x^2}$$

$$u = 3x^2$$

$$u' = 6x$$

$$\frac{d}{dx}(e^{2x-1}) = e^{2x-1} \cdot 2 = 2e^{2x-1}$$

$$\frac{d}{dx}(e^{-3/x}) = \frac{d}{dx}(e^{-3x^{-1}}) = e^{-3x^{-1}} \cdot 3x^{-2} = \frac{3e^{-3/x}}{x^2}$$

Find relative extrema of $f(x) = xe^x$

①

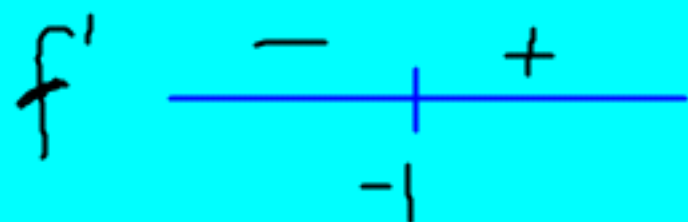
① CR, #S

② # line

③ id. extrema

② $f'(x) = x \cdot e^x + e^x \cdot 1 = 0$
 $e^x(x+1) = 0$

$$e^x = 0 \quad x+1 = 0$$
$$\emptyset \quad x = -1$$



$-2e^{-2} + e^{-2}$ ③
 $\frac{-2}{e^2} + \frac{1}{e^2} = -$

min @ $(-1, \frac{-1}{e})$

b/c $f'(x)$ changes from $-$ to $+$ @ this pt. (or f changes from dec. to inc.)

$f(-1) = -1 \cdot e^{-1}$ or $-\frac{1}{e}$

See ex 6 on page 344

S.T.Q.!!
😊

Integration of e^x

$$\int e^x = e^x + c \quad (\text{tough one, uh?!})$$

$$\int e^u = e^u + c$$

$$\text{ex: } \int e^{3x+1} dx = \int e^u \frac{du}{3} = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + c = \frac{1}{3} e^{3x+1} + c$$

$$u = 3x + 1$$

$$\frac{du}{dx} = 3$$

$$dx = \frac{du}{3}$$

$$dx = \frac{du}{3}$$

ck with
GUT!

undo: $\int 5xe^{-x^2} dx = 5 \int x e^u \cdot \frac{du}{-2x} = -\frac{5}{2} \int e^u du = -\frac{5}{2} e^u + C$

$$u = -x^2$$

$$\frac{du}{dx} = -2x$$

$$dx = \frac{du}{-2x}$$

$$= -\frac{5}{2} e^{-x^2} + C$$

Can ✓ by
differentiation!

$$\int \frac{e^{1/x} dx}{x^2} = \int \frac{e^u}{x^2} \cdot -x^2 du = -\int e^u du = -e^u + c = -e^{1/x} + c$$

$$u = \frac{1}{x} = x^{-1}$$

$$\frac{du}{dx} = -1x^{-2} = -\frac{1}{x^2}$$

$$dx = -x^2 du$$

$$\int \sin x e^{\cos x} dx$$

too easy!

Let's get DEFINITE!!

(Change upper & lower limits!

$$\int_0^1 e^{-x} dx = -\int_0^{-1} e^u du = -e^u \Big|_0^{-1} = -e^{-1} - -e^0 = \boxed{\frac{-1}{e} + 1}$$

$$u = -x$$

$$\frac{du}{dx} = -1 \quad u(0) = 0$$

$$dx = -du \quad u(1) = -1$$

$$\int_0^1 \frac{e^x}{1+e^x} dx = \int_2^{1+e} \frac{e^x}{u} \cdot \frac{du}{e^x} = \int_2^{1+e} \frac{1}{u} du = \ln|u| \Big|_2^{1+e} = \ln(1+e) - \ln 2 = \ln \frac{1+e}{2}$$

$$u = 1 + e^x$$


$$\frac{du}{dx} = e^x$$

$$dx = \frac{du}{e^x}$$

$$u(0) = 1 + 1 = 2$$

$$u(1) = 1 + e$$

change limits



$$\int_{-1}^0 e^x \cos e^x dx = \int_{1/e}^1 e^x \cdot \cos u \cdot \frac{du}{e^x} = \int_{1/e}^1 \cos u du = \sin u \Big|_{1/e}^1$$

$$u = e^x$$

$$\frac{du}{dx} = e^x$$

$$dx = \frac{du}{e^x}$$

$$= \sin 1 - \sin(1/e) = .482$$

see graphs of these last examples on pg 346

review directions in hw