Lesson 5.4---Exponential Functions Differentiation & Integration

NOTE: The first 2 pages of this lesson is precal...you're on you own with that!

$$\frac{d}{dx}(e^x) = e^x$$

NOTE: Proof of this is on page 343

$$\frac{d}{dx}(e^u) = e^u \cdot u'$$

$$\frac{d}{dx}(e^{3x^2}) = e^{3x^2} \cdot 6x = 6xe^{3x^2}$$

$u = 3x^2$ $\quad$ $u' = 6x$

$$\frac{d}{dx}(e^{2x-1}) = e^{2x-1} \cdot 2 = 2e^{2x-1}$$

$$\frac{d}{dx}(e^{-3/x}) = \frac{d}{dx}\left(e^{-3x^{-1}}\right) = e^{-3x^{-1}} \cdot 3x^{-2} = \frac{3e^{-3/x}}{x^2}$$

Find relative extrema of $f(x) = xe^x$

1

① CR. #s
② # line
③ id. extrema

2

$$f'(x) = x \cdot e^x + e^x \cdot 1 = 0$$

$$e^x(x+1) = 0$$

$$e^x = 0 \qquad x+1 = 0$$

$$\varnothing \qquad x = -1$$

f': $-$ on the left of -1, $+$ on the right of -1

$$-2e^{-2} + e^{-2}$$

$$\frac{-2}{e^2} + \frac{1}{e^2} = -$$

3

min @ $\left(-1, \frac{-1}{e}\right)$ b/c $f'(x)$ changes from - to + @ this pt. (or f changes from dec. to inc.)

$$f(-1) = -1 \cdot e^{-1} \text{ or } \frac{-1}{e}$$

See ex 6 on page 344

S.T.Q. !!

Integration of e^x

$\int e^x = e^x + c$ (tough one, uh?!)

$\int e^u = e^u + c$

ex: $\int e^{3x+1}dx = \int e^u \frac{du}{3} = \frac{1}{3}\int e^u du = 1/3e^u + c = 1/3e^{3x+1} + c$

$u = 3x + 1$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

ck with GUT!

udo:

$$\int 5xe^{-x^2}dx = 5\int x e^u \cdot \frac{du}{-2x} = -\frac{5}{2}\int e^u du = -\frac{5}{2}e^u + c$$

$u = -x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$$= -\frac{5}{2}e^{-x^2} + c$$

Can ✓ by differentiation!

$$\int \frac{e^{1/x}}{x^2}dx = \int \frac{e^u}{x^2}\cdot -x^2 du = -\int e^u du = -e^u + c = -e^{1/x} + c$$

$u = \frac{1}{x} = x^{-1}$

$\frac{du}{dx} = -1x^{-2} = \frac{-1}{x^2}$

$dx = -x^2 du$

$$\int \sin x \, e^{\cos x} dx$$

too easy!

Let's get DEFINITE!! (Change upper & lower limits!

$$\int_0^1 e^{-x}dx = -\int_0^{-1} e^u du = -e^u\Big|_0^{-1} = -e^{-1} - -e^0 = \boxed{\frac{-1}{e}+1}$$

$u=-x$

$\frac{du}{dx}=-1$ $u(0)=0$

$u(1)=-1$

$dx=-du$

$$\int_0^1 \frac{e^x}{1+e^x}\,dx = \int_2^{1+e} \frac{e^x}{u} \cdot \frac{du}{e^x} = \int_2^{1+e} \frac{1}{u}\,du = \ln|u|\Big|_2^{1+e} = \ln(1+e) - \ln 2 = \ln\frac{1+e}{2}$$

$u = 1 + e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$u(0)=1+1=2$

$u(1) = 1 + e$

change limits

$$\int_{-1}^{0} e^x \cos e^x \, dx = \int_{1/e}^{1} e^x \cdot \cos u \cdot \frac{du}{e^x} = \int_{1/e}^{1} \cos u \, du = \sin u \Big|_{1/e}^{1}$$

$$= \sin 1 - \sin(1/e) = .482$$

$u = e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

see graphs of these last examples on pg 346 review directions in hw