AP CAL Lesson 3.3--Increasing & Decreasing Functions & The First Derivative Test

Lesson by video

THM 3.5

When f(x) is increasing, f '(x)>0   (+)
When f(x) is decreasing, f '(x)<0 (-)

Use the PIND principle

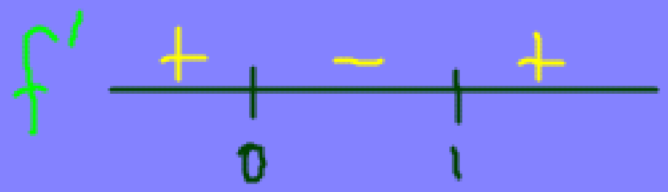
When given f(x), to determine intervals of increasing or decreasing:
(1) Determine the critical #s
(2) Set up a # line using the critical #s (& #s not in Domain) to test f '(x) in intervals
(3) determine the sign of f '(x) in each interval
(4) Decide if f(x) is inc or dec using the PIND principle

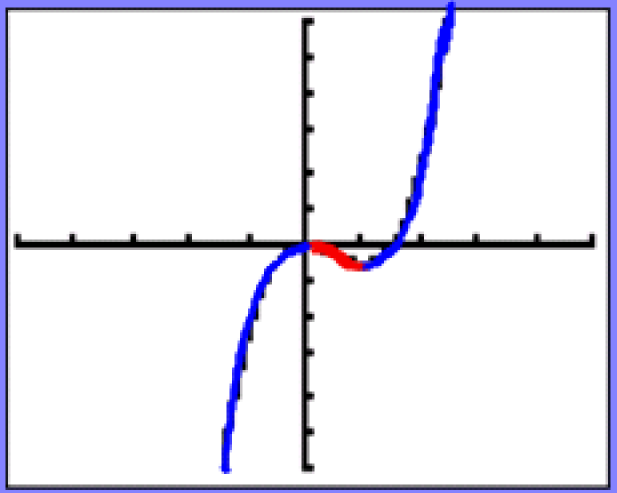
examples....

Determine the intervals on which f(x) is increasing & decreasing:

ex: $f(x) = x^3 - 3/2x^2$

$f'(x) = 3x^2 - 3x = 0$ @ $x = 0$ and $x = 1$ (critical #s)





test #s in each interval to determine sign of f '

Picture the t. lines on each interval...are their slopes pos. or neg.?

f inc: $(-\infty, 0)$ & $(1, \infty)$

f dec: $(0, 1)$

b/c f ' <0

b/c f ' >0

ck with TI-89!

The First Derivative Test (see p 176 for formal defn)

If c is a critical # of f then f(x) is a relative max if f '(x) changes from pos. to neg.
or f(c) is a relative min if f '(x) changes from neg. to pos.

NOTE: If the sign of f '(x) DOESN'T change, then f(c) is neither a max or a min!

SO, the first derivative test helps us determine maxs and mins of a function!

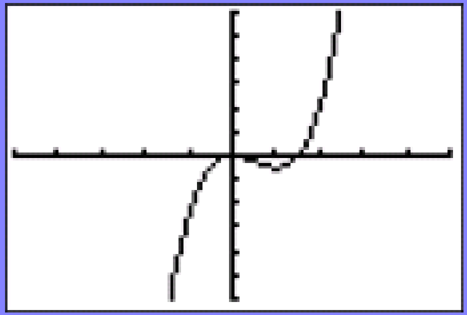
same prblm from before

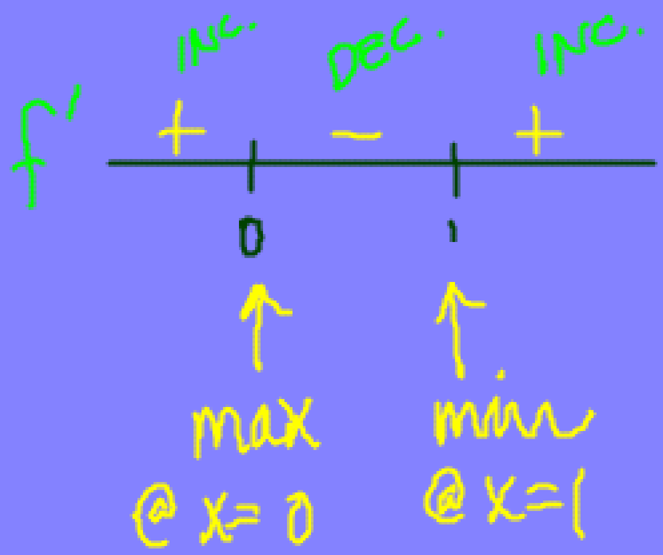
$f(x) = x^3 - 3/2x^2$

$f'(x) = 3x^2 - 3x = 0$

@ $x = 0$ and $x = 1$ (critical #s)

possible extrema!





MUST STATE WHY

Max @ x = 0 b/c f ' changes from pos. to neg. @ x = 0
Min @ x = 1 b/c f ' changes from neg. to pos. at x = 1

max pt: (0, ?); min pt: (1, ?)

UDO: $f(x) = \frac{1}{2}x - \sin x \quad [0, 2\pi]$

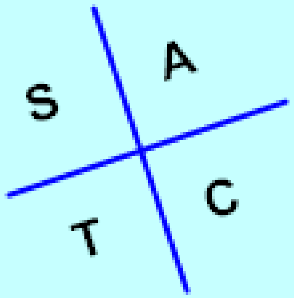
Tell where f(x) is increasing, decreasing and list any extrema

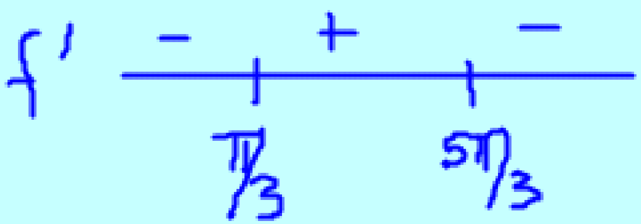
$f'(x) = \frac{1}{2} - \cos x$

$\frac{1}{2} - \cos x = 0$

$\frac{1}{2} = \cos x$

@ $x = \pi/3$ & $5\pi/3$





INC: $(\pi/3, 5\pi/3)$

DEC: $(0, \pi/3)$, $(5\pi/3, 2\pi)$

EXTREMA:

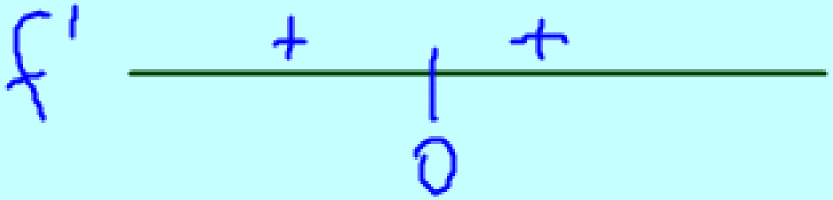
max: $(5\pi/3, \; 5\pi/6 + \sqrt{3}/2)$

min: $(\pi/3, \; \pi/6 - \sqrt{3}/2)$

Let's ck with TI-89!
GRAPH f(x) in y1

An example of a function with no extrema; $f(x) = x^3$

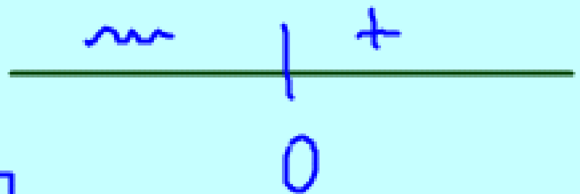
$f'(x) = 3x^2 = 0$ @ $x = 0$ (cr #)



f is inc $(-\infty, \infty)$

another ex: $f(x) = \sqrt{x}$

$f'(x) = \dfrac{1}{2\sqrt{x}}$



f is inc. everywhere in domain

Only cr # is x = 0

Can't test points to left of 0 b/c -#s not in domain of f '(x)
So, f(x) is increasing everywhere!

UDO: $f(x) = (x^2 - 4)^{2/3}$ Determine intervals of inc/dec as well as extrema

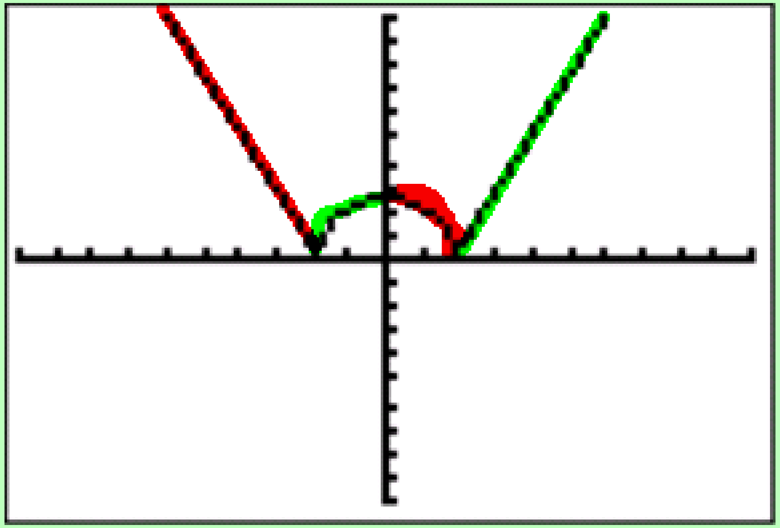
use GUT do differentiate & test values in f '(x)!

$$f'(x) = \frac{4x}{3(x^2-4)^{1/3}}$$

f '(x) = 0 @ x = 0

f '(x) DNE @ x = ±2

} cr. #s



f '(x): - (−∞, −2); + (−2, 0); - (0, 2); + (2, ∞)

inc: (-2,0) & (2,∞)

dec: (-∞, -2) & (0,2)

Why?

Max: $(0, 4^{2/3})$; Min: (-2, 0) & (2,0) Why?

RECALL: c is NOT a cr # if it is not in the domain of the original function

Find cr #s, then check D of f to add to test intervals any values where f is indefined

You must use cr #s AND undefined values to set up test intervals

remember: cr#s don't always produce min/max

see ex 4 in txt

$$f(x) = \frac{x^4 + 1}{x^2}$$

$$f'(x) = \frac{2x^4 - 2}{x^3}$$

$f'(x) = 0$ @ ± 1

$f'(x)$ DNE @ $x = 0$ BUT $x = 0$ is NOT a cr. # b/c it is not in the domain of $f(x)$!

HOWEVER, it should still be used when testing intervals on the # line!

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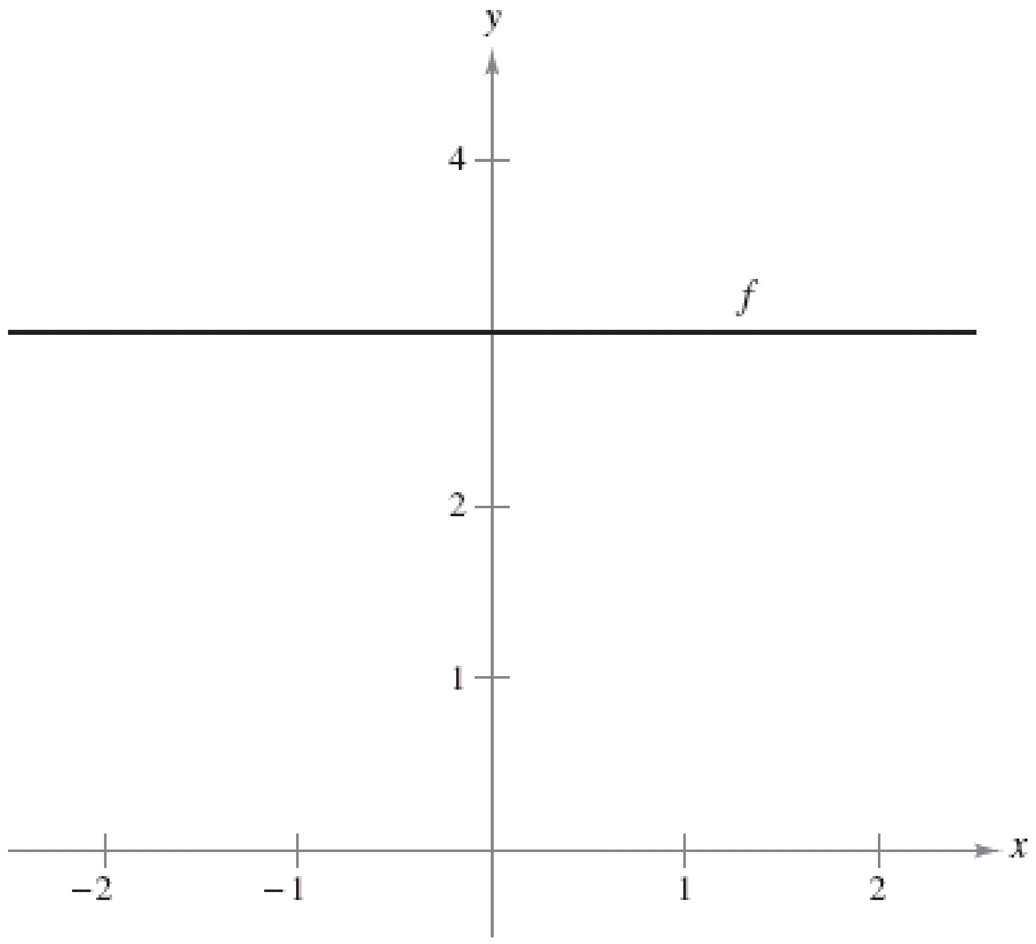
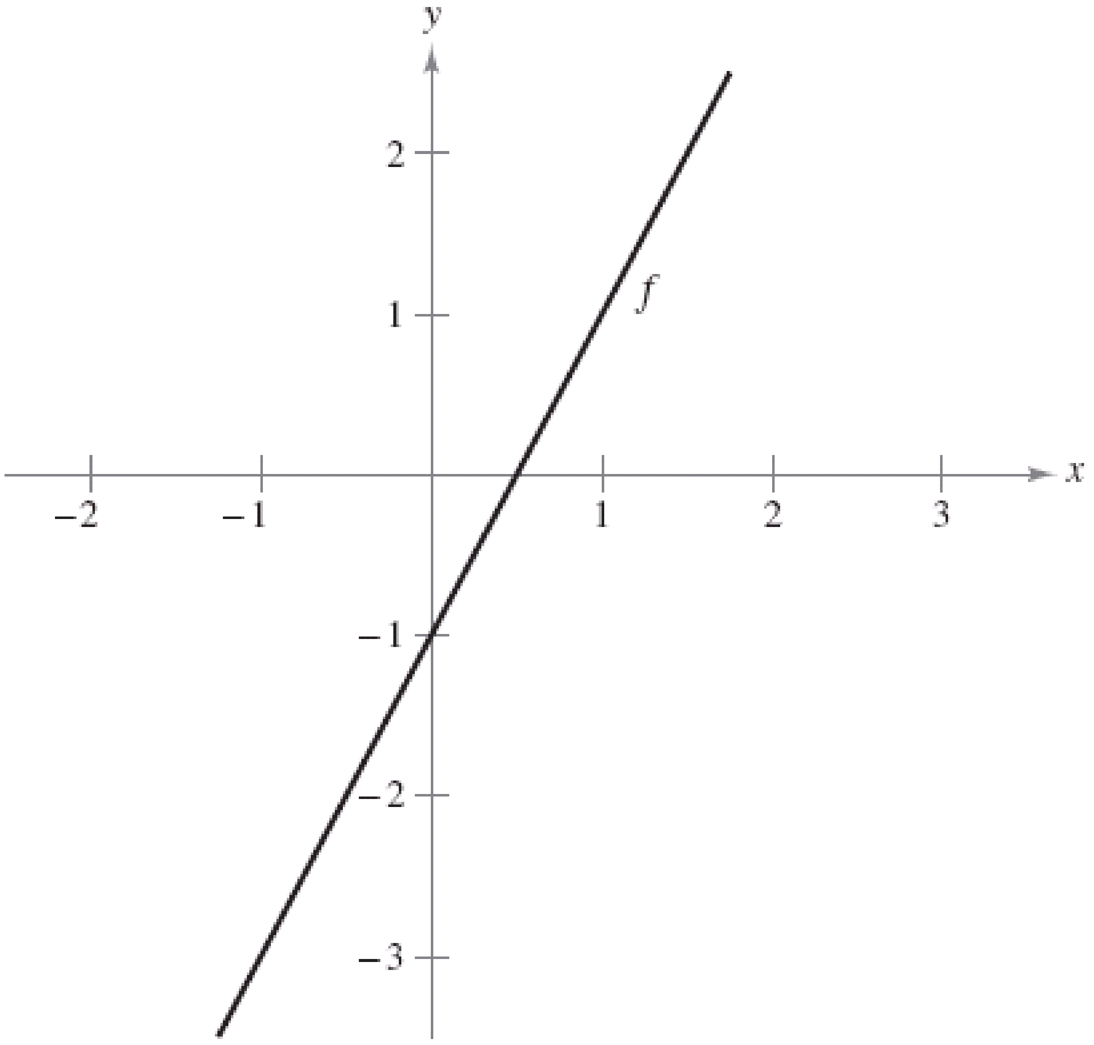
The graph of f is shown in the figure. Sketch a graph of the derivative of f.



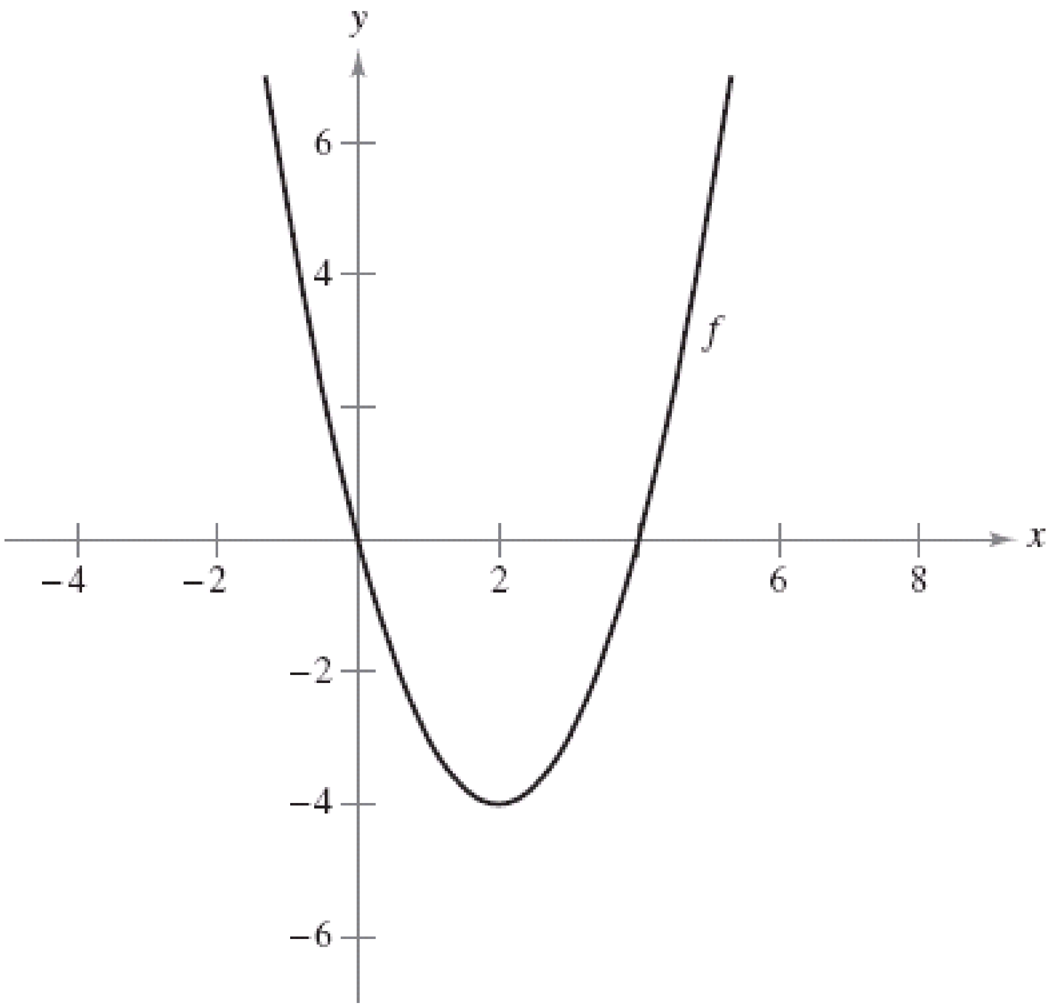
3.3/ # 44

The graph of f is shown in the figure. Sketch a graph of the derivative of f.



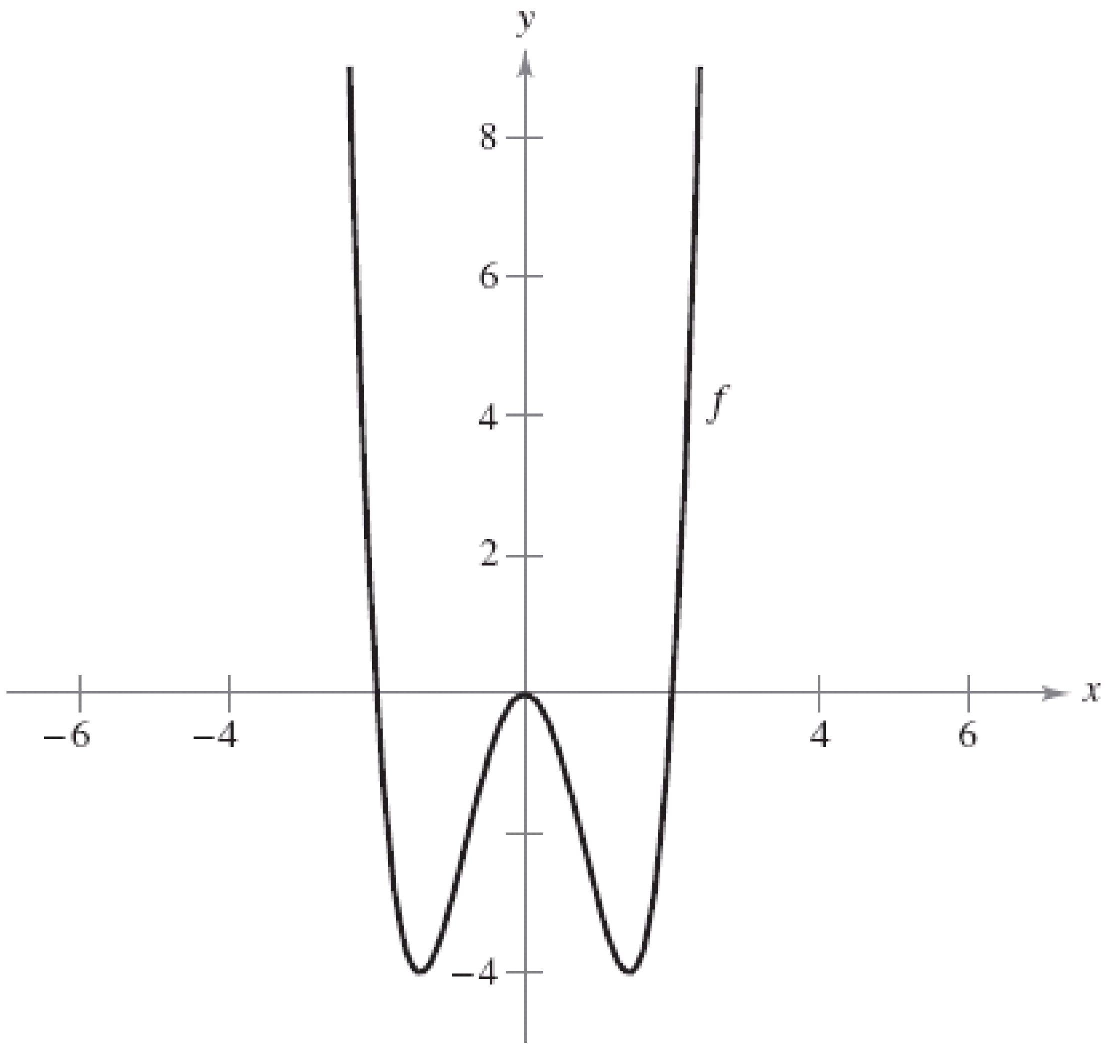
3.3/ # 45

The graph of f is shown in the figure. Sketch a graph of the derivative of f.



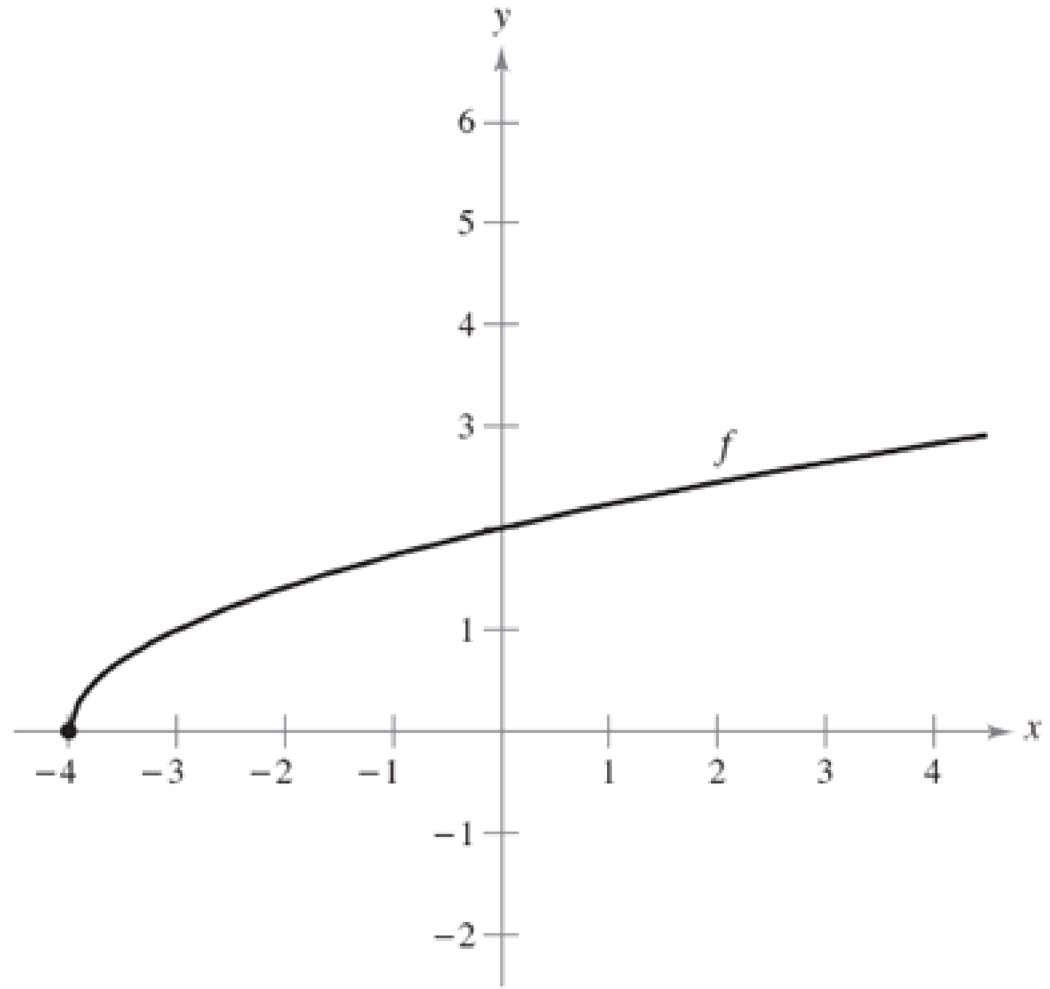
3.3/ # 46

The graph of f is shown in the figure. Sketch a graph of the derivative of f.



3.3/ # 47

The graph of f is shown in the figure. Sketch a graph of the derivative of f.



3.3/ # 48

The graph of f is shown in the figure. Sketch a graph of the derivative of f.